

Examen final

**Exercice1: (10pts=3pts+4pts+3pts)**

Calculer ce qui suit :

1.  $I_1 = \int x^5 \ln x dx$  (Intégration par parties)
2.  $I_2 = \int \frac{x-1}{x^2+2x+3} dx$  (Élément simple de type 3)
3.  $I_3 = \int_{-1}^{+1} (1+x^3)^4 x^2 dx$  (Changement de variable)

**Exercice2: (10pts=4pts+3pts+3pts)**

Résoudre les équations différentielles suivantes :

1.  $y^2 + (x+1)y' = 0$  (Equation du premier ordre à variables séparables)
2.  $y'' + y = 0$  (Equation du deuxième ordre sans second membre)
3.  $y'' - y = 0$  (Equation du deuxième ordre sans second membre)

Exercice 1

•  $I_1 = \int x^5 \ln x \, dx$  (par parties)  $f = \ln x$   $f' = \frac{1}{x}$   
 $g' = x^5$   $g = \frac{1}{6} x^6$

$$I_1 = \frac{1}{6} x^6 \ln x - \int \frac{1}{6} x^6 \cdot \frac{1}{x} dx = \frac{1}{6} x^6 \ln x - \frac{1}{6} \int x^5 dx$$

$$I_1 = \frac{1}{6} x^6 \ln x - \frac{1}{6} \cdot \frac{1}{6} x^6 + C = \frac{1}{6} x^6 \left[ \ln x - \frac{1}{6} \right] + C \quad (C \in \mathbb{R})$$

(03 pts)

•  $I_2 = \int \frac{x-1}{x^2+2x+3} dx$   $x^2+2x+3=0$   $\Delta = 4-12 = -8 < 0$ .

$$I_2 = \int \frac{\frac{1}{2}(2x+2) - 2}{x^2+2x+3} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x+3} dx - 2 \int \frac{1}{x^2+2x+3} dx$$

$$= \frac{1}{2} \ln(x^2+2x+3) - 2J.$$

$$J = \int \frac{1}{x^2+2x+3} dx = \int \frac{1}{(x+1)^2+2} dx = \frac{1}{2} \int \frac{1}{\left(\frac{x+1}{2}\right)^2+1} dx$$

$$= \frac{1}{2} \int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} dx \quad \text{on pose } t = \frac{x+1}{\sqrt{2}} \Rightarrow dt = \frac{1}{\sqrt{2}} dx \Rightarrow dx = \sqrt{2} dt.$$

$$= \frac{\sqrt{2}}{2} \int \frac{1}{t^2+1} dt = \frac{\sqrt{2}}{2} \operatorname{arctg} t = \frac{\sqrt{2}}{2} \operatorname{arctg} \left( \frac{x+1}{\sqrt{2}} \right)$$

$$I_2 = \frac{1}{2} \ln(x^2+2x+3) - \sqrt{2} \operatorname{arctg} \left( \frac{x+1}{\sqrt{2}} \right) + C \quad (C \in \mathbb{R})$$

(04 pts)

•  $I_3 = \int_{-1}^{+1} (1+x^3)^4 x^2 dx$  on pose  $t = (1+x^3) \Rightarrow dt = 3x^2 dx$   
 $x = -1 \Rightarrow t = 0$ ,  $x = 1 \Rightarrow t = 2$ .

$$= \frac{1}{3} \int_0^2 t^4 dt = \frac{1}{3} \left[ \frac{1}{5} t^5 \right]_0^2 = \frac{1}{3} \times \frac{1}{5} \times 2^5$$

$$I_3 = \frac{32}{15}$$

(03 pts)

Exercice 2:

1.  $y^2 + (x+1)y' = 0 \Rightarrow (x+1)y' = -y^2 \Rightarrow -\frac{y'}{y^2} = \frac{1}{x+1}$  ( $y \neq 0$  solution)

on pose  $y' = \frac{dy}{dx}$  ainsi on obtient  $-\frac{dy}{y^2} = \frac{dx}{x+1}$

$\int -\frac{dy}{y^2} = \int \frac{dx}{x+1} \Rightarrow +\frac{1}{y} = \ln|x+1| + C \quad C \in \mathbb{R}$

$\Rightarrow y = \frac{1}{\ln|x+1| + C}$  et  $y=0$  (4pts)

2.  $y'' + y = 0$  Eq caractéristique  $r^2 + 1 = 0 \Rightarrow r^2 = -1 \Rightarrow r^2 = i^2$   
 $\Rightarrow r = 0 \pm i$

$y_0 = (C_1 \cos x + C_2 \sin x) e^{0x} \Rightarrow y_0 = C_1 \cos x + C_2 \sin x$   
 $C_1, C_2 \in \mathbb{R}$  (3pts)

3.  $y'' - y = 0$  Eq caractéristique  $r^2 - 1 = 0 \Rightarrow r = 1$  ou  $r = -1$

$y_0 = C_1 e^x + C_2 e^{-x}$  (3pts)